

# Using a hybrid approach to model central carbon metabolism across the cell cycle

Cécile Moulin <sup>1,2</sup>, Laurent Tournier <sup>2</sup> and Sabine Peres <sup>1,2</sup>

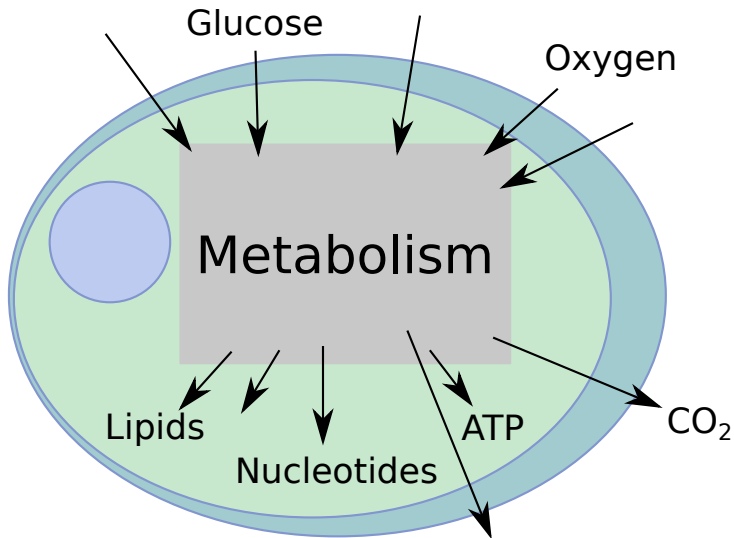
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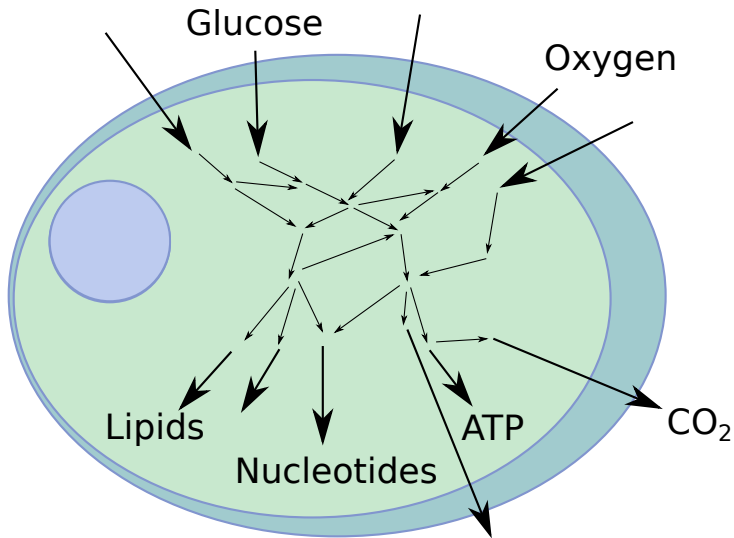
6th April 2019 - HSB 2019



## Metabolism is a production system

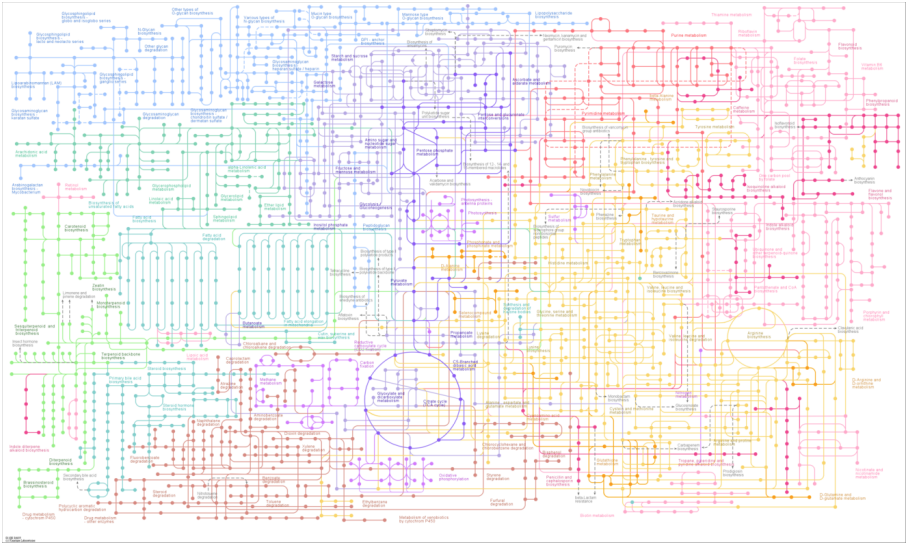


# Metabolism is a reaction network

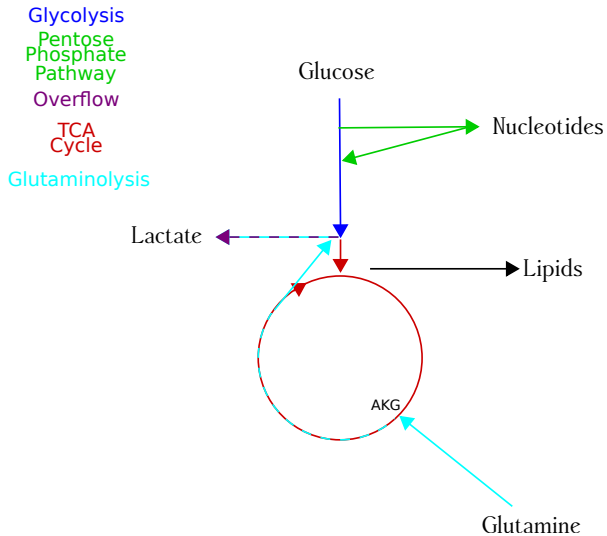


# Metabolism is a huge network

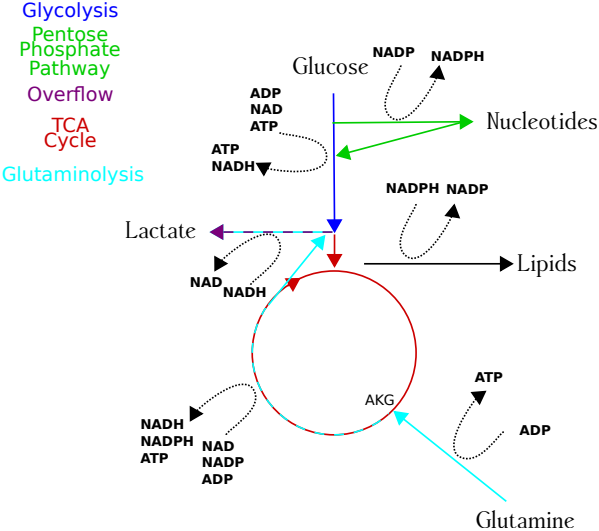
Recon3D:13 543 metabolic reactions, 4 140 unique metabolites (Brunk et al., 2018).  
Kegg Map (metabolism global view):



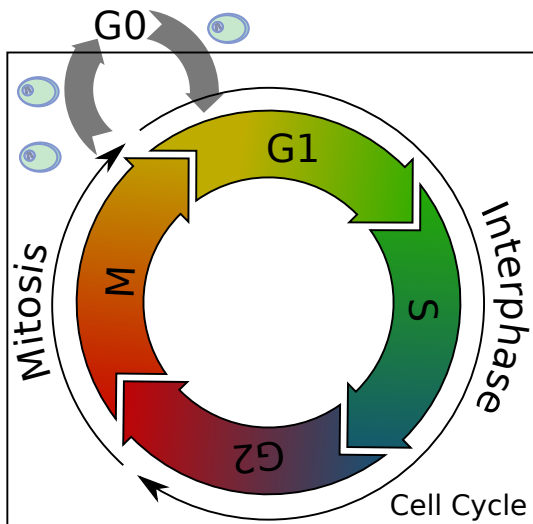
# Solution: Metabolic Pathways



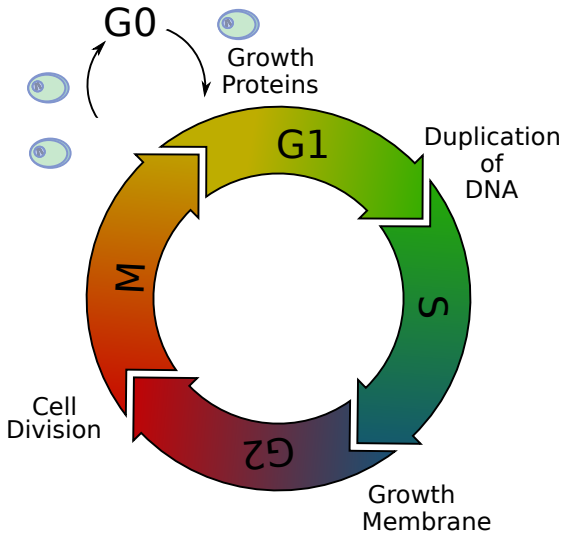
# Metabolic pathways are coupled by currency metabolites



# Eukaryotic cell cycle: From one mother cell to two daughter cells

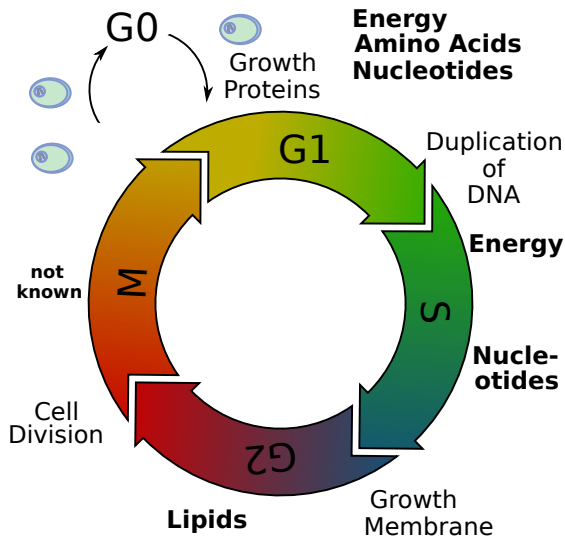


## Eukaryotic cell cycle: Divided into 4 phases



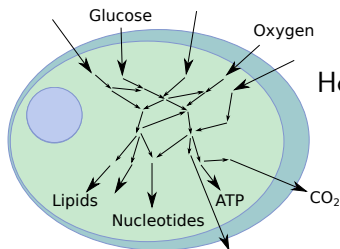


# Eukaryotic cell cycle: Linked to the metabolism

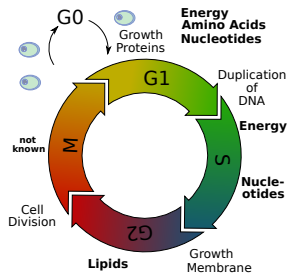


(da Veiga Moreira et al., Theoretical Biology and Medical Modelling, 2015)

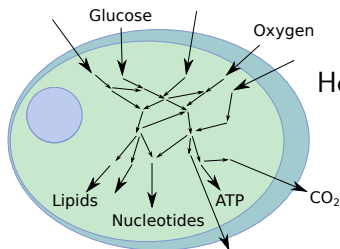
# Goal: Create a model coupling metabolism and cell cycle



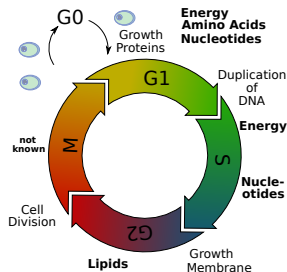
How do metabolism and cell cycle communicate?



# Goal: Create a model coupling metabolism and cell cycle



How do metabolism and cell cycle communicate?



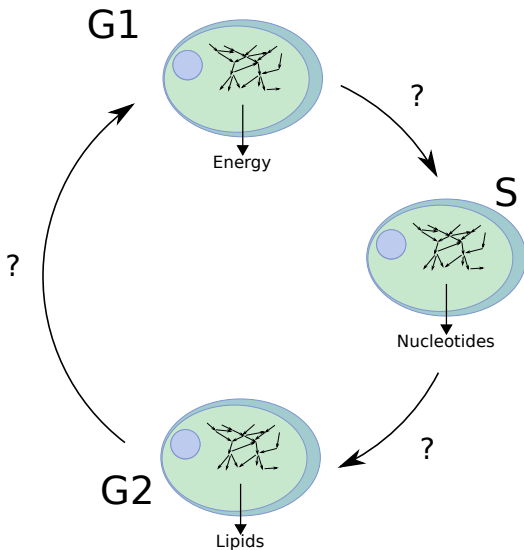
Challenges:

Different time scales

Which level of knowledge?

“Wherever continuous and discrete dynamics interact, hybrid systems arise.”

(Heemels *et al.*, Handbook of Hybrid Systems Control, 2009)



# Outline

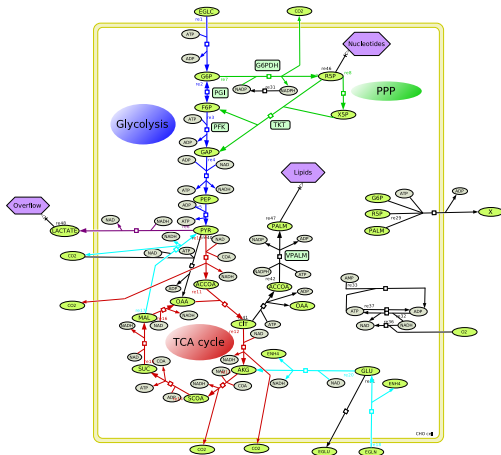
- 1 The metabolic model
  - Introduction of the model
  - Test the model with cell cycle inputs
- 2 The hybrid model
  - Parameters selection
  - Behavior of the hybrid model

# Extended mammalian Central Carbon Metabolism model

The original models:

Robitaille *et al.*, PLOS ONE, 2015  
(CHO)

da Veiga Moreira *et al.*, Scientific Reports, 2019  
(mice)

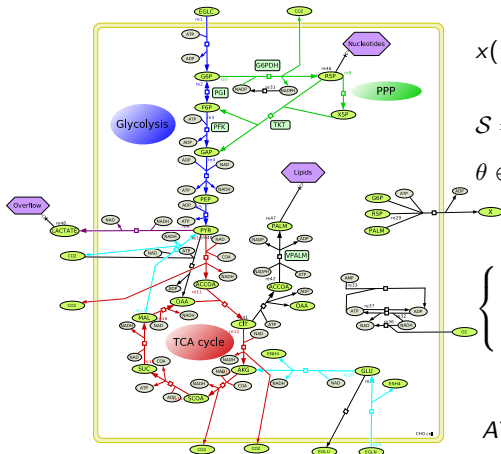


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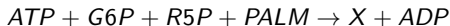


$$x(t) = \begin{pmatrix} x_I(t) \\ x_{II}(t) \\ x_{biomass}(t) \end{pmatrix} \begin{matrix} 16 \\ 7 \\ 1 \end{matrix}$$

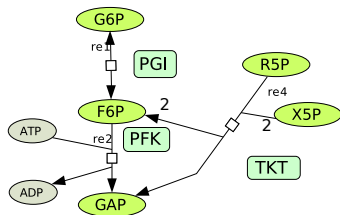
$$S = \begin{pmatrix} S_I \\ S_{II} \end{pmatrix} \in \mathbb{Q}^{23 \times 30}$$

$$\theta \in \mathbb{R}_+^{100}$$

$$\begin{cases} \dot{x}_I & = S_I \nu_\theta(x(t)) - \mu_\theta(x(t)) x_I(t), \\ \dot{x}_{II} & = S_{II} \nu_\theta(x(t)), \\ \dot{x}_{biomass} & = \mu_\theta(x(t)) x_{biomass}(t). \end{cases}$$



## Example: production/consumption of fructose 6-phosphate (F6P)

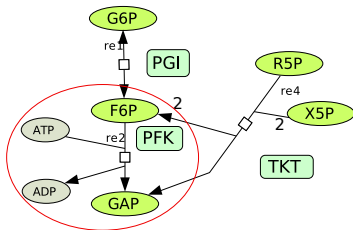


$$\frac{dF6P}{dt} = \nu_{pgi\_f}(t) - \nu_{pgi\_b}(t) - \nu_{pfk}(t) + 2\nu_{tkk}(t) - \mu(t)F6P(t).$$

(Robitaille *et al.*, PLOS ONE, 2015)



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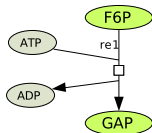


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# Example: production/consumption of fructose 6-phosphate (F6P)

PFK



1 Michaelis-Menten

$$\nu_{max} = k_{cat}[E]$$

$$\nu_{pfk} = \nu_{max} \frac{F6P}{K_{m1} + F6P}$$

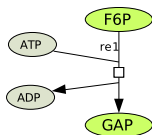
(Robitaille *et al.*, PLOS ONE, 2015)

(Ghorbaniaghdam *et al.*, Bioprocess and Biosystems Engineering, 2013)

(Segel, Enzyme kinetics, 1993)

# Example: production/consumption of fructose 6-phosphate (F6P)

PFK



- 1 Michaelis-Menten  
 $\nu_{max} = k_{cat}[E]$
- 2 Currency Metabolites

$$\nu_{pfk} = \nu_{max} \frac{F6P}{K_{m1} + F6P} \frac{\frac{ATP}{ADP}}{K_{m2} + \frac{ATP}{ADP}}$$

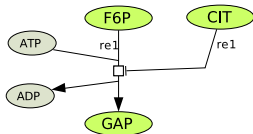
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# Example: production/consumption of fructose 6-phosphate (F6P)

PFK



- 1 Michaelis-Menten  
 $\nu_{max} = k_{cat}[E]$
- 2 Currency Metabolites
- 3 Non-competitive inhibition

$$\nu_{pfk} = \nu_{max} \frac{F6P}{K_{m1} + F6P} \frac{\frac{ATP}{ADP}}{K_{m2} + \frac{ATP}{ADP}} \frac{K_i}{K_i + CIT}$$

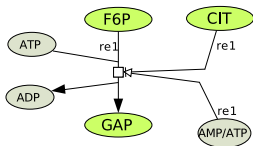
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# Example: production/consumption of fructose 6-phosphate (F6P)

PFK



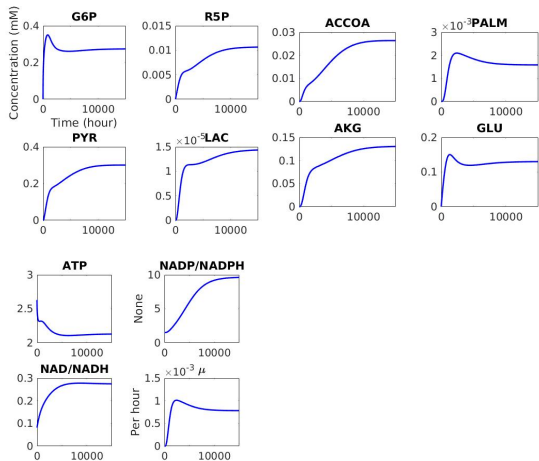
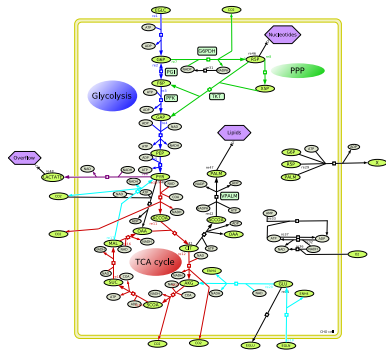
- 1 Michaelis-Menten  
 $v_{max} = k_{cat}[E]$
- 2 Currency Metabolites
- 3 Non-competitive inhibition
- 4 Non-essential activation

$$v_{pfk} = v_{max} \frac{F6P \left(1 + \frac{\beta}{\alpha K} \frac{AMP}{ATP}\right)}{K_{m1} \left(1 + \frac{1}{K} \frac{AMP}{ATP}\right) + F6P \left(1 + \frac{1}{\alpha K} \frac{AMP}{ATP}\right)} \frac{\frac{ATP}{ADP}}{K_{m2} + \frac{ATP}{ADP}} \frac{K_i}{K_i + CIT}$$

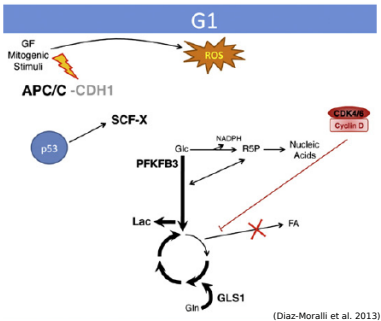
(Robitaille et al., PLOS ONE, 2015)  
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# The model reaches a stationary regime

$$\theta \in \mathbb{R}_+^{\sim 100}$$



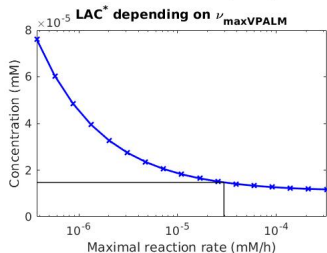
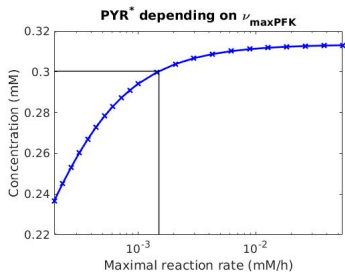
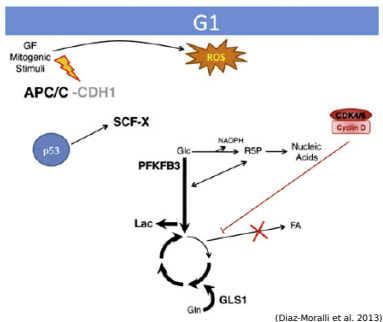
# The model responds correctly to G1 inputs



[...] the accumulation of PFK/FB3 leads to the activation of glycolysis and an increase in lactate production. [...] Moreover cyclin D1 is able to downregulate the expression of lipogenic enzymes [...] preventing pyruvate consumption in lipogenesis and contributing to lactate formation.

(Diaz-Moralli et al. Pharmacology & Therapeutics, 2013)

# The model responds correctly to G1 inputs

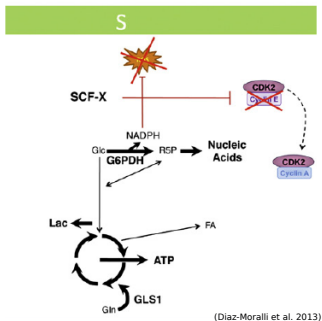


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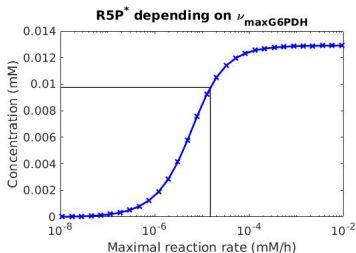
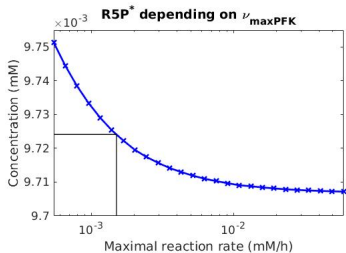


# The model responds correctly to S inputs

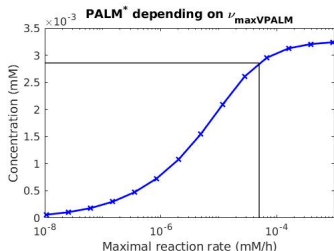
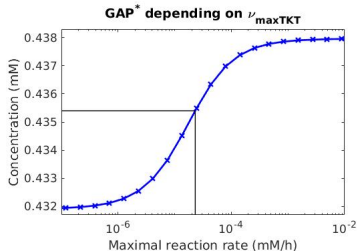
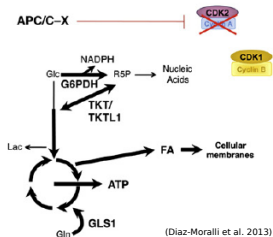


[...] proliferating cells increase G6PD activity during late G1- and S-phases [...]. Moreover, during S-phase the activation of the SCF ubiquitin ligase [...] allows [the proteasome degradation of] PFKFB3. [...] Through these mechanisms cells redirect the glucose flux from the direct glycolytic pathway to the PPP [...]

(Diaz-Moralli et al. Pharmacology & Therapeutics, 2013)



# The model responds correctly to G2 inputs



[...]transketolase activity showed an acute increase in late S. This shift allows [...] recycling the excess of R5P back to glycolysis in late S- and G2-phases, when lipid synthesis [...] is highly demanded]. Moreover, [...] the activation of transcription of lipogenic enzymes [contributes] to this process [...].

(Diaz-Moralli et al. Pharmacology & Therapeutics, 2013)

# Creation of the three sub-models: G1, S, G2

G1 ↗ PFK: ↗ glycolysis

G1 ↘ VPALM: ↗ LAC

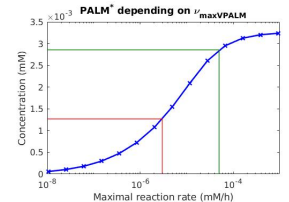
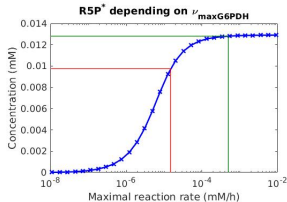
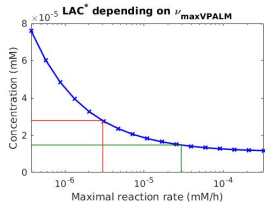
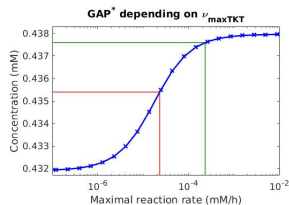
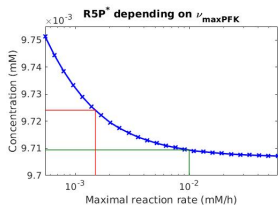
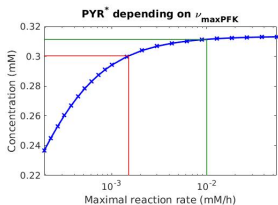
S ↘ PFK: ↗ PPP

S ↗ G6PDH: ↗ PPP

G2 ↗ TKT: ↗ end of PPP

G2 ↗ VPALM: ↗ PALM

	G1	S	G2
PFK	+	-	-
G6PDH	-	+	+
TKT	-	-	+
VPALM	-	-	+

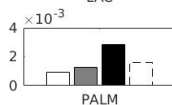
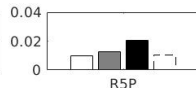
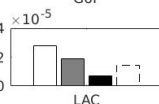
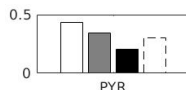
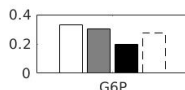
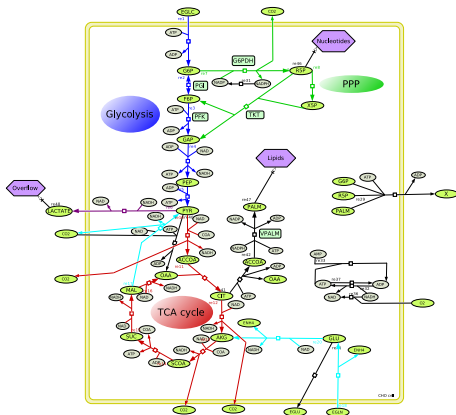


# Validation of the sub-models

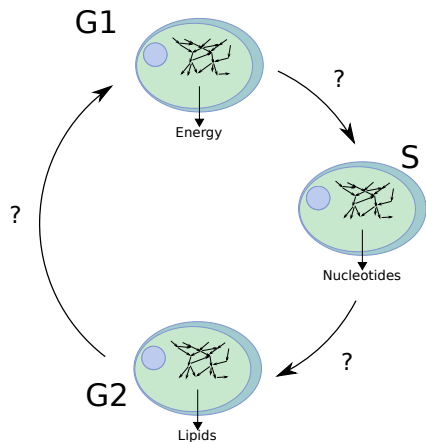
G1: High Glycolysis activity and Lactate production ✓

S: High beginning of Pentose Phosphate Pathway activity ✓

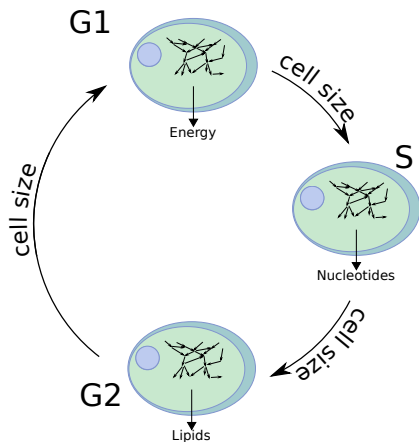
G2: High Pentose Phosphate Pathway activity and Lipids production ✓



# Hybrid model: transition between phases



# Hybrid model: transition between phases



## Rules

G1 ends when:

$$x_{biomass}(t) := (1 + \alpha)x_b^0$$

S ends when:

$$x_{biomass}(t) := (1 + \beta)x_b^0$$

G2 ends when:

$$x_{biomass}(t) := (1 + \gamma)x_b^0$$

New parameters:

$$\alpha, \beta, \gamma := 1$$

# Validation of the hybrid model

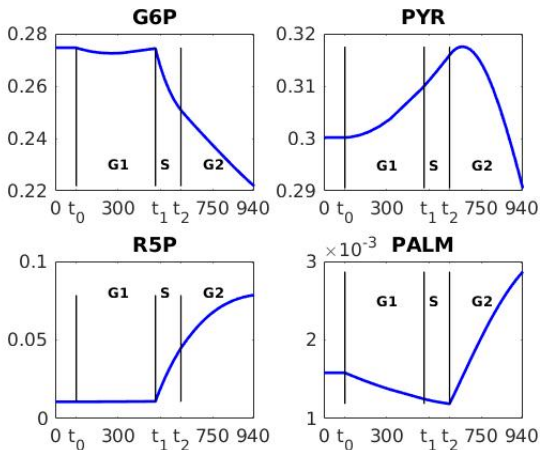
Duration of phases ×

G1: High Glycolysis activity ✓

S: High beginning of Pentose Phosphate Pathway activity ✓



G2: High Pentose Phosphate Pathway activity and Lipids production ✓

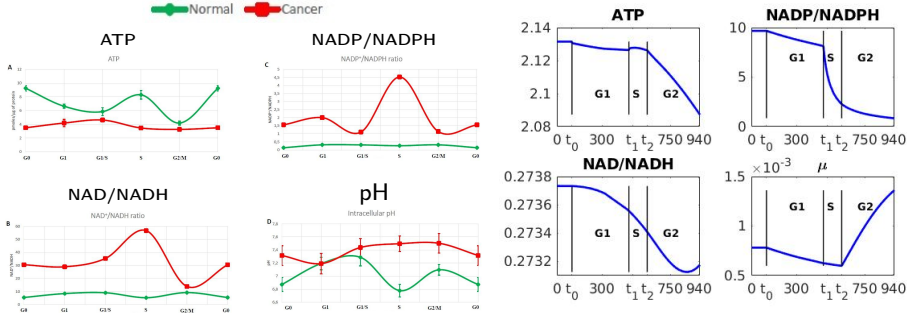
$x_b^0 = 1.1 \cdot 10^{-4} L$ ,  $\alpha = 0.3$ ,  $\beta = 0.4$



# Comparison with experimental data

NAD/NADH variations  $\times$   
 NADP/NADPH decrease in S  $\checkmark$   
 ATP variations  $\checkmark$

 Normal 
  Cancer



(da Veiga Moreira *et al.*, *Metabolites*, 2016)

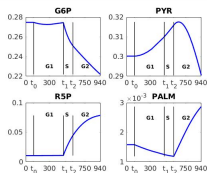


# Conclusion and Perspectives

From a metabolic model to a **hybrid model**, representing **metabolism through cell cycle**.

Succession of phases:

- G1: Glycolysis activity
- S: Nucleotides production
- G2: Lipids production



Toward the understanding of the **interconnections** between **cell cycle** and **metabolism**

- Are these changes minimum ?
- Other areas of the Central Carbon Metabolism (Amino-Acids)
- Change biomass function (supply and demand)
- Adaptations to cancer metabolism